Frustrated Total Reflection

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Abstract

In the following report we analyzed the phenomena of transmission and reflection of microwaves through glass, acrylic glass and metal. In order to measured the capacity of this materials, we determined the irradiance transmitted and reflected with a pair of transmitter and receiver of microwaves. Also we examined the phenomenon of frustrated total internal reflection due to two prisms separated by a thin layer of air. We determined the intensity of the transmitted evanescent wave through the prisms in function of the distance between them. Finally, we made a parallelism between frustrated total internal reflection and the tunneling phenomenon described by quantum mechanics.

1 Frustrated total internal reflection

When light is incident in the interface between two different media part of that light is reflected back to the first medium and the other part is transmitted to the second one. If, however, the index of refraction of the first medium (n_1) is greater than the one corresponding to the second medium (n_2) and the angle of incidence exceeds the critical angle, then total internal refraction occurs. In this case, all the light is reflected [4]. When analyzing the surface that acts as a boundary for both media, there is a small, exponentially decaying wave in the second material denominated evanescent wave. Thus, if a material with a high index of refraction is put next to the first medium boundary, it can pick up this wave causing a phenomenon called frustrated total internal reflection (FTIR). When FTIR happens, the energy on the reflected wave diminishes in the quantity carried by the evanescent wave. Otherwise, if frustration does not occur, all energy returns to the reflected wave so it does not experience energy loss [1].

1.1 Experimental setup for FTIR

The experimental setup used to produce FTIR is shown in Figure 1:

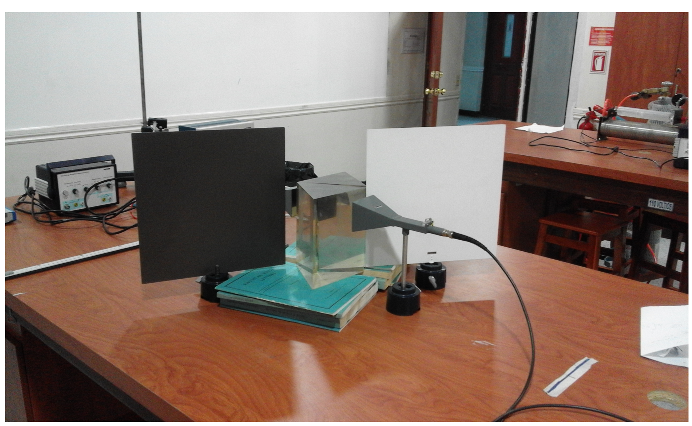

Figure 1: Experimental set up for FTIR

For the experiment we used a couple of transmitter and receiver of microwaves. We put between both of the devices two prisms of synthetic resin. We put the transmitter right after one of the prism, aligning the edge of the funnel with the edge of the prism and then we cover it from behind. We did exactly the same with the receiver and the other prism in the opposite side (as shown in Figure 1).

For executing the experiment, we separated both prisms making the distance between them going from 1 [mm] to 2 [cm] and we measured the voltage generated in the receiver (that is proportional to the received irradiance).

1.2 Theoretical considerations about FTIR

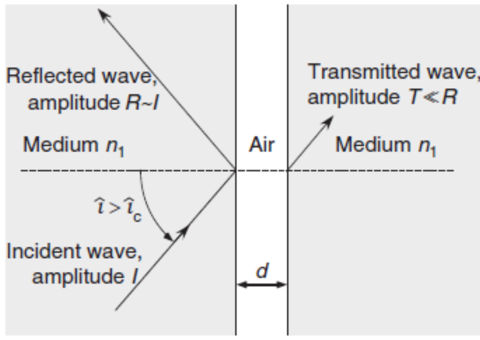

Figure 2: Experiment to show FTIR

One of the simplest ways to generate FTIR is establishing a system similar to the one sketched in Figure 2. Two materials of a high index of refraction separated by a thin layer of a different material (in our case the medium number 1 was resin (prisms) and the 2 was air). Assuming that the waves are plane waves, the transmitted wave (in medium 2) can be modeled as:

$$\vec{E_2} = \vec{E_0}exp[i(\vec{k}\cdot\vec{r} - wt)] = \vec{E_0}exp[i(k_2 sin\theta_2 x + k_2 cos\theta_2 y - wt)] \tag{1}$$

From Snell's law is possible to establish that:

$$\frac{n_2}{n_1} = \frac{1}{n_r} = \frac{sin\theta_2}{sin\theta_i} \tag{2}$$

For θ_i the angle of incidence, θ_2 the angle of transmission in medium number 2 and n_1 and n_2 the indexes of refraction of each medium. For a evanescent wave in the second medium, it can be formally established that [2]:

$$sin\theta_2 = n_r sin\theta_i = (1+\beta^2)^{\frac{1}{2}} > 1 \tag{3}$$

$$cos\theta_2 = (1 - sin^2\theta_2)^{\frac{1}{2}} = \pm i\beta \tag{4}$$

In equations 3 and 4 we have that $\beta > 0$ and in particular for equation 4 we chose the upper sign due to the fact that the transmitted wave propagates to the right of the system. Then, combining equations 1, 3 and4 is possible to write down the spatial dependence of the transmitted wave like [2]:

$$\vec{E_2} = \vec{E_0}exp(-i\gamma y)exp[ix(k_2^2 + \gamma^2)^{\frac{1}{2}}] \tag{5}$$

In equation 5 $\gamma = k_2\beta$ and it is called attenuation coefficient. This equation represents that the wave is evanescent and decays exponentially to zero as $y \to \infty.\gamma$ also has a value equivalent to:

$$\gamma = k_2\sqrt{n_r^2 sin^2\theta_i - 1} \tag{6}$$

On the other hand, it is possible to show that for the general case of the three layer problem established in Figure 2 the expression of the transmission coefficient is [4]:

$$T = \left[\alpha sinh^2(\gamma dy) + 1\right]^{-1} \tag{7}$$

Where the constant α can take two different values depending if we are analyzing the transmittance perpendicular or parallel to the incidence plane. The values are respectively [4]:

$$\alpha_\perp = \left[\frac{(n_r^2 - 1)}{2n_r}\right]^2 \left\{\frac{1}{[cos^2\theta_i(n_r^2 sin^2\theta_i - 1)]}\right\} \tag{8}$$

$$\alpha_{\parallel} = \alpha_{\perp}[(n_r^2 + 1)sin^2\theta_1 - 1]^2 \tag{9}$$

For big values of d (the distance between both media of the same index, in our case the prisms) the expression in 7 can be simplified to:

$$I_{trans} = I_0 exp(-2\gamma d) \tag{10}$$

1.3 Results for FTIR

1.3.1 Determination of the coefficient of attenuation

Distance (d)	Voltage	Distance (d)	Voltage
[mm]	[mV]	[mm]	[mV]
±0.05	±3	±0.05	±3
1	317	11	6
2	290	12	2
3	238	13	1
4	221	14	0
5	169	15	0
6	114	16	0
7	73	17	0
8	43	18	0
9	35	19	0
10	20	20	0

Table 1: Experimental result for the intensity transmitted in function of the distance between prisms

In Table 1 is present the experimental information that relates the distance between the prisms with the intensity transmitted by the evanescent wave. Considering that equation 10 works for big values of d and also that it was extremely difficult to measured the distances corresponding of 1 and 2 [mm], we decided to neglect that information. With all the previous considerations, we used a exponential fitting with the experimental data. The result is presented in Figure 3.

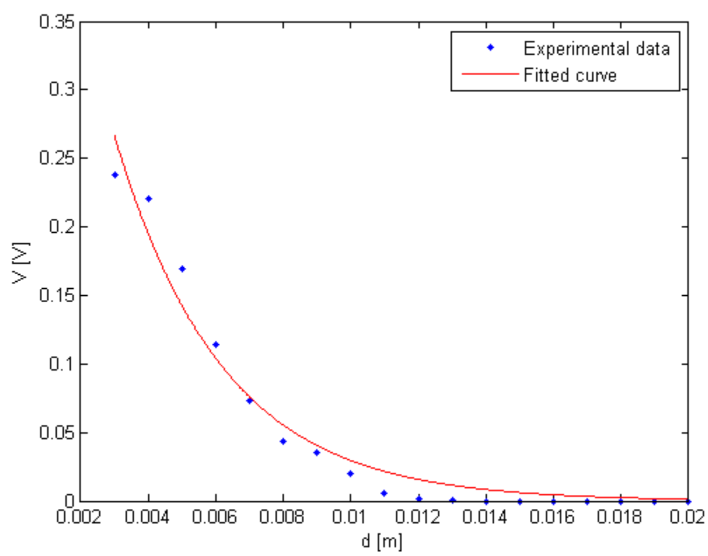

Figure 3: Exponential fitting

The curve presented in Figure 3 has the form $f(x) = a \times exp(b \times x)$ where $a = 0.684 \pm 0.151$ and $b = -314 \pm 50$. Comparing this relation with equation 3 we found that $\gamma = 1.57 \pm 0.25$ [cm^{-1}] and $I_0 = 684 \pm 151$ [mV].

When comparing the experimental attenuation coefficient with the theoretical one equivalent to $\gamma_{theo} = 1.64$ [cm^{-1}] the error is 4.2%. This error shows that the method used to measure the attenuation coefficient has high accuracy. But, the uncertainty obtained from the exponential fitting causes the reduction of the precision of the method.

On the other hand, the value obtained for I_0does not match exactly with the value measured when $d = 0$ (we measured $I = 329$ [mV]). This fact does not mean that the experimental value is wrong. The prisms had considerably irregular sides, then it was nos possible to measured correctly this value. Moreover, the value of voltage given by the receiver without prisms was closer to our experimental value (it was closer to 500 mV).

Finally, our results are a experimental prove of the presence of evanescent waves between the prisms, which are catch by the second prism causing the "frustration" of total internal reflection. This fact can be observed in Figure 3. When the second prism is enough close to the first one, it causes that the evanescent wave takes energy from the reflected wave. As the prisms is put further, the energy on the evanescent wave decays exponentially until the system fulfill again the conditions for total internal reflection.

The fact that a small portion of transmitted wave can exist even though total internal reflection is present can allow us to make a parallelism between this phenomena and the tunneling phenomena trough a square barrier in one dimension. This phenomena is described by quantum mechanics.The coefficient of transmission due to a square barrier of height U (potential energy) that goes from 0 to L for a particle with energy E is [3]:

$$T(E) = \left\{ 1 + \frac{1}{4} \left[\frac{U^2}{E(U-E)} \right] sinh \xi L \right\}^{-1} \tag{11}$$

$$\xi = \left(\frac{2m(U-E)}{\hbar^2} \right)^{\frac{1}{2}} \tag{12}$$

Comparing this equations with equation 7 and the coefficient $\alpha_\perp$, we can make the correspondence $\frac{mE}{\hbar^2} \rightarrow \frac{n_r^2}{4\lambda^2}$ and $\frac{mU}{\hbar^2} \rightarrow \frac{n_r^2-1}{2\lambda^2}$. Thus, for $\theta_i = 45$ degrees, both relations are the same [4]. Both cases represents similar (but no equal situation), the presence of something after a boundary. FTIR represents the presence of evanescent waves even though there is total frustrated reflection. In contrast, the tunneling phenomena represents the probability of the particle to go trough the barrier. Moreover, both cases have different dimensions. The barrier penetration is unidimensional, meanwhile the FTIR is two dimensional and also depends on the polarization of the incident wave.

1.3.2 Determination of the index of refraction of the synthetic resin (prism)

Using equation 6 with $\theta_i \approx 60$ degrees, $n_{air} = 1$ and $k_2 = \frac{2\pi f}{c} = 1.98$ [cm] ($f = 9.45$ [GHz]) we found $n_{resin} = 1.47 \pm 0.11$. When compared with the theoretical value of $n_{theo} = 1.50$ the error is 2%.

2 Transmission and reflection

Propagation characteristics of different materials are necessary to determine in order to know which material is useful for a given function. An example is crystal, a material with a high index of transmission, it is used to manufacture lens. Another example is crystalline silicon, it has a relatively high index of absorption, this material is present in solar panels.

2.1 Transmission and reflection characteristics of glass, acrylic glass and metal

Consider a plane electromagnetic wave that travels through a medium with index of refraction n_1 and then through another medium with index n_2 ($n_1 \neq n_2$). Lets suppose both mediums are homogeneous such that scattering due to material flaw is negligible. Since theres a medium change, transmission, reflection and absorption occur.

If I_1 is the total irradiance, we can define the transmission coefficient T, reflection coefficient R and absorption coefficient A as the ratio between the corresponding partial irradiance and the total irradiance, i.e.

$$J = \frac{I_J}{I_1} \tag{13}$$

where $J = T$, R or A. Due to conservation of energy, the following expression is true:

$$T + R + A = 1 \tag{14}$$

T, R and A depends on the angle of incidence as well as of the electronic and atomic characteristics of the material.

In the case of metals, valence electrons move freely within the material. The incident electromagnetic wave excites free electrons causing them to oscillate parallel to the electric field. Electrons now accelerated produce an opposite phase wave which has the same frequency. In transmission, this produces a destructive interference between the primary and secondary waves. Now is evident why most of the incident wave is reflected. Collisions between electrons and lattice disturbances cause part of the incident radiation energy to be converted to Joule heat, which means that it is absorbed. If wavelength is comparable with atomic lattice distances, the previous model has some corrections.

We know that non conductors and dielectric materials have no free charge carriers. Nevertheless, if theres an electric field, charge transfers occur in the material. A reason is the presence of a permanent dipole momenta, and another is an induced dipole momentum, i.e. a shifting of the valence electrons in relation to the molecular body, whose restoring forces F can be described as elastic in a first order approximation:

$$\vec{F} \sim -\vec{r} \tag{15}$$

The frequency dependence of the transmission, reflection and absorption characteristics of dielectric materials is due to the dependence on the exciting waves frequency of the relation between the phase shift of the forced oscillations of a damped harmonic oscillator and the incident waves.

The materials used in this part of the practical lab work are three plates of different types of glass (a circular thin plate, a thin rectangular plate and a thick plate), a metal plate, a Plexiglas plate and a plastic plate.

The system set up to determine the reflection characteristics of each material is shown in Figure 4.

Figure 4: System setup, determination of reflection characteristics of metal and dielectrics

First, the reflector voltage of the klystron has to be modulated in “ext” and “$\sim$”, such that is adjusted to maximum output. Place the microwave transmitter and receiver one next to the other. The distance between both antennas and the material plate has to be $40\,[cm]$, this to get the maximum reflection signal. The irradiance reflected is measured using the multimeter.

The system set up to determine the transmission characteristics of each material is shown in Figure 5.

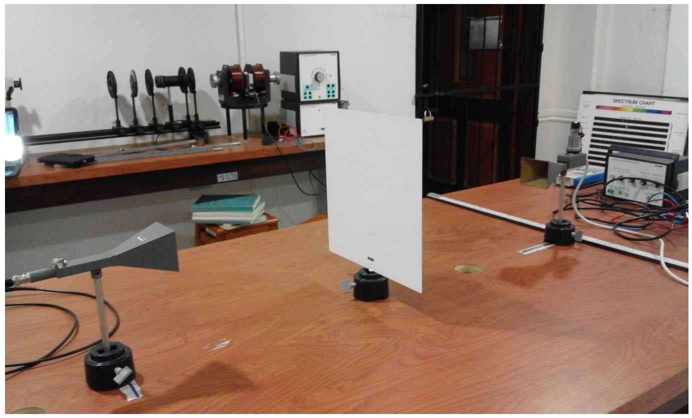

Figure 5: System set up, determination of transmission characteristics of metal and dielectrics

Antennas are placed in the same axis facing each other, the distance between them has to be 80 $[cm]$, this to compensate the loss of irradiance in air due to absorption by water molecules and to the divergence of the emitted microwaves. The material plate is placed in the middle of both antennas. Irradiance transmitted through air is also measured.

As we know, radiation intensity is proportional to its voltage, so

$$\frac{I}{I_1} = \frac{V}{V_1} \tag{16}$$

The total irradiance I_1 is considered to be the one with no plate between both antennas.

Transmission, reflection and absorption coefficients determined are shown in Table 2.

Materials	Reflection	Transmission	1- Reflection - Transmission
Air	0±0.00005	1±0.003	0±0.00005
Metal	0.513±0.002	0±0.00005	0.487±0.002
Circular thin glass	0.31±0.05	0.4±0.1	0.3±0.2
Thin glass	0.09±0.03	0.28±0.05	0.63±0.08
Thick glass	0±0.00005	0.83±0.03	0.17±0.03
Plexiglas	0.006±0.003	0.67±0.05	0.33±0.05
Plastic	0.004±0.003	0.78±0.03	0.21±0.03

Table 2: Reflection, transmission and 1-reflection-transmission measurements of different probes

As we can note in Table 2, and as we already said, the transmitted intensity through air is considered as the initial intensity. Metal's characteristics show there is no a substantial difference between the coefficient of reflection and the coefficient of absorption.

Observe that there are differences between transmission and reflection characteristics of the different types of glass. One circular thin glass has 0.31±0.05 as coefficient of reflection while the thick one has 0±0.00005. Both also show a difference between their coefficient of transmission. Another data that draws attention is the coefficient of absorption of the thin glass, 0.63±0.08, this because the difference with the coefficients of absorption of the other types of glass is relatively high.

Plexiglas transmission coefficient is 0.67±0.05, take into account that this material is similar to glass. The plastic screen shows a relatively high coefficient of transmission, it is important to say that this screen was physically more like to the metal plate than to a glass plate.

Ambient conditions possibly affected the information obtained, such as the presence of air as a medium that also has transmission and reflection characteristics.

3 Conclusions

In the case of the FTIR the experimental results showed the existence of evanescent waves. The fact that we could measure the intensity of a wave after the second prism is an experimental probe that even though the conditions of

total internal reflection are fulfill, it is possible that a small portion of the wave to go through the boundary that separates the two interfaces. The result of FTIR is a wave that travels through the second prism and it decays exponentially with the distance between both prisms. Also, we could establish the coefficient of attenuation of the wave and the index of refraction (of the resin prism) in a pretty accurate way, but not totally precise due to the high uncertainty derived from the exponential fit used.

On the other hand, for the analysis of the transmission and reflection through different material, even when it comes from the same material, transmission and reflection characteristics for each sample does not repeat. This suggests that the different compositions and mass densities can cause this contrast between the sample. Note that absorption phenomenon always occur, but the quantitative information obtained is not enough accurate to show this.

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